

STA 119 Worksheet on Confidence Intervals and Hypothesis Testing

Section 1: Confidence Intervals

Problem 1

The average amount of excess waste from 16 cans of juice was measured and found to be 3.25 ounces, with a population standard deviation of 0.3 ounces.

1. Determine a 95% confidence interval for the population mean.

$t \Rightarrow$
 $n=16 \Rightarrow$
 $\alpha=0.025 \Rightarrow$
 $\frac{\alpha}{2}=0.0125 \Rightarrow$
 $t_{\alpha/2, n-1} = 2.131$
 $\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$
 $3.25 \pm 2.131 \frac{0.3}{\sqrt{16}}$

2. Determine a 98% confidence interval for the population mean.

$n=16$
 $\alpha=0.025$
 $\Rightarrow \frac{16}{2} = 8$
 $\alpha=0.01$
 $\chi^2_{\alpha/2, n-1} = 27.4884$
 $\chi^2_{1-\alpha/2, n-1} =$
 $1-\alpha=0.98$
 $\alpha=0.02$
 $\chi^2_{0.025, 15} = 24.739$
 $\chi^2_{0.975, 15} = 6.757$
 $\frac{0.3}{\sqrt{16}}$

3. Determine a 99% confidence interval for the population mean.

Problem 2

The average score of 250 students was calculated as 78, with a population standard deviation of 1.75.

1. Determine a 95% confidence interval for the population mean.

250
 78
 1.75
 $z_{0.025} = 1.96$
 $z_{0.005} = 2.58$
 $z_{\alpha/2} \Rightarrow 1.96$
 $(\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}})$

2. Determine a 98% confidence interval for the population mean.

1.75

3. Determine a 99% confidence interval for the population mean.

$$\left(\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}} \right)$$

Section 2: Determining Sample Size

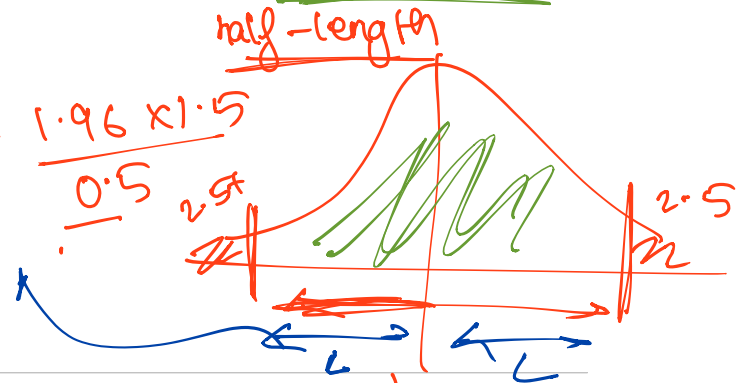
Problem 1

To calculate a 95% confidence interval for the population mean with a population standard deviation of 1.5, determine the sample size required if the desired margin of error is 0.5.

$$L = z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$\Rightarrow 0.5 = 1.96 \times \frac{1.5}{\sqrt{n}}$$

$$\Rightarrow \sqrt{n} = \frac{1.96 \times 1.5}{0.5}$$



Problem 2

Calculate the sample size necessary to achieve a 95% confidence interval for the mean, assuming a population standard deviation of 1.0 and a margin of error of 0.25.

$$\alpha = 0.05 \quad \alpha/2 = 0.025 \quad \text{half length}$$

$$\Rightarrow 0.25 = 1.96 \times \frac{1.0}{\sqrt{n}}$$

$$n = 8$$

Section 3: Confidence Interval for Population Mean

Problem 1

The average baking time for 10 cookies at a bakery is 20 minutes, with a sample standard deviation of 1.25 minutes.

1. Determine a 95% confidence interval for the population mean.

2. Determine a 99% confidence interval for the population mean.

Problem 2

A doctor measured the average blood sugar level in 25 patients with a particular disease, finding a mean of 124 and a standard deviation of 2.5.

1. Determine a 95% confidence interval for the population mean.

2. Determine a 99% confidence interval for the population mean.

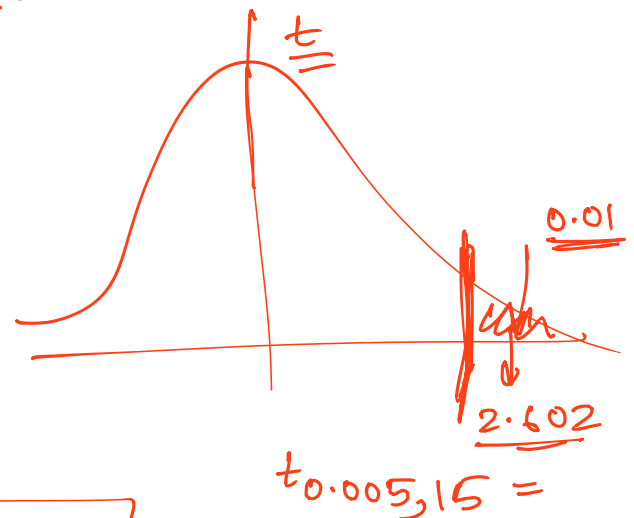
3. Determine a 98% confidence interval for the population mean.

Section 4: Hypothesis Testing

Problem 1

Company A claims its tires last 7,000 miles longer than those of Company B. For Company B, the known population mean is 30,000 miles. Company A's data is $n=16$, $\bar{x}=40,000$, and $s=4,000$ s. Test this claim at the 0.01 significance level.

$$\Rightarrow \begin{aligned} H_0: \mu_A &= 37000 \rightarrow \mu_0 \\ H_a: \mu_A &> 37000 \end{aligned}$$



$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{40000 - 37000}{4000/\sqrt{16}}$$

$$t \Rightarrow 3$$

$$\Rightarrow \frac{3000}{4000} \times 4$$

Problem 2

The average flight time on a specific path is 10.5 hours. A new route is being tested to reduce this time. For a sample of 15 flights on the new path, the average time is 9.75 hours, with a standard deviation of 0.5 hours. Test at the 0.05 significance level whether the new path reduces the flight time.